Problem of the Week
Problem C
Step by Step

Debit and credit cards contain account numbers which consist of many digits. When purchasing items online, you are often asked to type in your account number. Because there are so many digits, it is easy to type the number incorrectly. The last digit of the number is a specially generated check digit which can be used to quickly verify the validity of the number. A common algorithm used for verifying numbers is called the *Luhn Algorithm*. A series of operations are performed on the number and a final result is produced. If the final result ends in zero, the number is valid. Otherwise, the number is invalid.

The steps performed in the Luhn Algorithm are outlined in the flowchart below. Two examples are provided.

Example 1:

Number: 135792

Reversal: 297531

$A = 2 + 7 + 3$
$\quad = 12$

$2 \times 9 = 18$
$2 \times 5 = 10$
$2 \times 1 = 2$

$B = (1 + 8) + (1 + 0) + 2$
$\quad = 9 + 1 + 2$
$\quad = 12$

$C = 12 + 12 = 24$

C does not end in zero.
The number is not valid.

Flowchart:

- Accept any number to check.
- Reverse the digits.
- Add all the digits in the odd positions. Call this A.
- Double each of the remaining digits.
- Add the digits in each of the products, and then find the sum of these numbers. Call this B.
- Calculate $C = A + B$.
- Does C end in 0?
  - NO → INVALID
  - YES → VALID

Example 2:

Number: 1357987
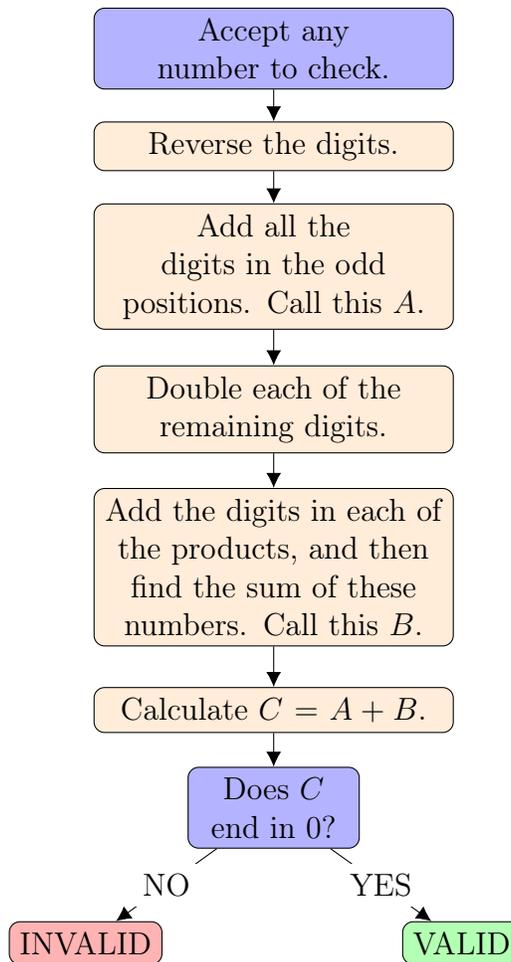
Reversal: 7897531

$A = 7 + 9 + 5 + 1$
$\quad = 22$

$2 \times 8 = 16$
$2 \times 7 = 14$
$2 \times 3 = 6$

$B = (1 + 6) + (1 + 4) + 6$
$\quad = 7 + 5 + 6$
$\quad = 18$

$C = 22 + 18 = 40$

C ends in zero.
The number is valid.

The number 1953 $R8T9$ 467 is a valid card number when verified by the Luhn Algorithm. R and T are each single digits of the number such that R is less than T.

Determine all possible values of R and T.

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